

RESEARCHING THE UNIT

AN ACTIVITY TO VISUALISE THE UNIT



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identifies some of the common errors students make when selecting and using units to measure area and perimeter.

According to the National Council of Teachers of Mathematics (NCTM) (1989, 2000) students at all levels should be exposed to geometry and measurement topics. Standards documents have identified the importance of primary school students selecting appropriate units for measurements in one- and two-dimensional space (NCTM 1989, 2000). This paper provides teachers with an overview of the research regarding students' understanding of one- and two-dimensional units and presents an activity that will help students develop a deeper understanding of the meaning of units in various contexts. This paper is guided by the following questions:

1. What does the existing literature say about students' difficulties with understanding the concept of units involved with various measurements?
2. What activities have been presented in the literature to provide teachers with tools to develop students' understanding of the unit?
3. Based on the literature, what is an activity that can be used to develop students' understanding of the unit?

The difficulty young students have with moving from linear units in one-dimensional space to square units in two-dimensional space has been acknowledged by the NCTM (2000, p. 45)

Understanding that different units are needed to measure different attributes is sometimes difficult for young children. Learning how to choose an appropriate unit is a major part of understanding measurement. For example, students in pre-kindergarten through grade 2 should learn that length can be measured using linear tools but area cannot be directly measured this way. Young children should see that to measure area they will need to use a unit of area such as a square region ...

Such an emphasis on the importance of selecting appropriate units has continued with the recent release of the *Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006). The purpose of this document is to emphasise “a small number of significant mathematical ‘targets’ for each grade level” (p. 6). The NCTM expects grade 4 students to use their understanding of linear units to develop understanding of measurements in two-dimensional space.

The van Hiele model of the development of geometric thinking can also be applied to the development of understanding of units. It describes a progression of hierarchical levels, each of which must be mastered before a student can move to the next. The levels are as follows:

- Level 0: Visualisation
- Level 1: Analysis
- Level 2: Informal deduction
- Level 3: Formal deduction
- Level 4: Rigour.

The concepts of units would be considered Level 1 topics, since students would have to analyse what one- and two-dimensional units represent and this is dependent upon their ability to visualise linear, square and cubic units.

Some of the existing literature that investigates measurement concepts focuses on measurements in one and two dimensions (e.g., Chapell & Thompson, 1999; Ferrer, Hunter, Irwin, Sheldon, Thompson & Vistro-

Yu, 2001; Gilliland, 2002; Sherman & Randolph, 2004). Although the activities presented in these articles mention the importance of selecting appropriate units for one- and two-dimensional measurements, they fail to elaborate on how students should choose appropriate units for a given measurement. In many cases the students are provided with the units and asked only to perform some type of calculation. Examples of these types of tasks are also common on the National Assessment of Educational Progress (NAEP).

Outhred and Mitchelmore (2000) argued that students must be exposed to situations where they need to move from one-dimensional to two- and three-dimensional units. The NCTM suggested using concrete materials to help students visualise the difference between units in multiple dimensions (1989, p. 5) but Outhred and Mitchelmore (2000) cautioned educators about developing these types of experiences: “First, concrete materials may conceal the very relationship they are intended to illustrate... Second, children may not relate the concrete materials to the mathematical concepts they are supposed to represent” (p. 146). Their concern is that students will focus on “covering” whatever object they are trying to measure and counting the number of objects they used, rather than focusing on the units involved with the measurements. Doig, Cheeseman, and Lindsay (1995) found that children develop a better understanding of one- and two-dimensional units by utilising wooden tiles rather than paper tiles to cover a rectangular surface. It is important to further investigate how the use of concrete materials (i.e., toothpicks, paper, wooden tiles, and wooden cubes) strengthens students’ understandings of one-, two-, and three-dimensional units.

NAEP items and student performance

Despite the expectations of the NCTM, results from the National Assessment of Educational Progress (NAEP) indicate that students have difficulties with these measurements. The NAEP involves students in grades 4, 8, and 12. The assessment consists of both multiple-choice and constructed-response questions and is administered every 4 years. The results are based on a representative sample of students in the United States rather than a census of all students at any one period of time.

Students' difficulties with area measurement are evident in responses such as that shown in Figure 1 to a grade 8 NAEP item. Of the students responding to this question, 48% answered incorrectly by calculating the perimeter rather than the area. This shows a lack of understanding of what the "square units" represent.

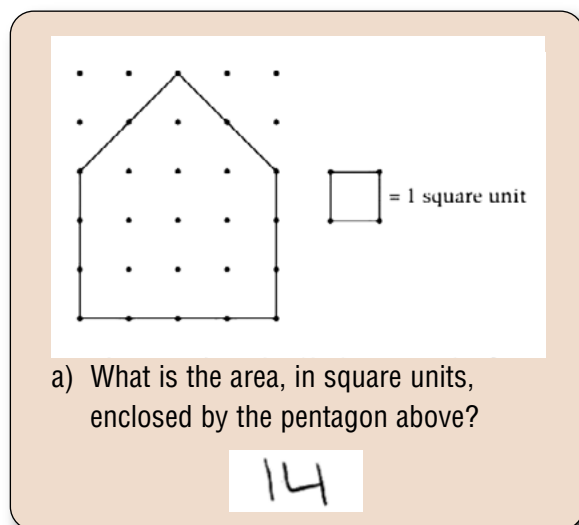


Figure 1. Student response to a grade 8 NAEP question.

Figure 2 illustrates a response to a grade 4 question that shows a similar confusion about the meaning of square units. Here, the student created a rectangle with a side length of 12 linear units rather than a rectangle with an area of 12 square units. This error was evident in the responses of 48% of the students who responded to the question.

The NCTM expects grade 5 students to build upon their understanding of one- and

5) On the grid below, draw a rectangle with an area of 12 square units

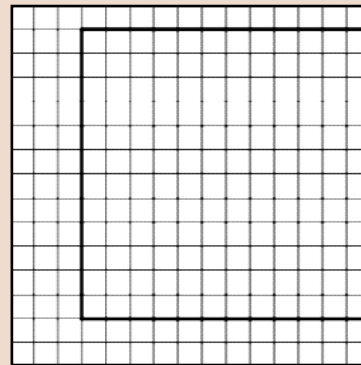


Figure 2. Grade 4 NAEP question.

two-dimensional measurements to develop understanding of the use of cubic units to measure volume, but this is also difficult for many students. Figure 3 shows an NAEP question, to which just 41% of grade 8 students and 55 % of grade 12 students responded correctly.

If a measurement of a rectangular box is given in 48 cubic inches, then the measurement represents the

- A distance around the top of the box
- B length of an edge of the box
- C surface area of the box
- D volume of the box

Figure 3. Grade 4 NAEP question.

Other NAEP items, such as that shown in Figure 4 that provide the units and require students to perform a calculation, do not require students to consider the selection of appropriate units.

The perimeter of a square is 36 inches.
What is the length of one side of the square?

- A 4 inches
- B 6 inches
- C 9 inches
- D 18 inches

Figure 4. Grade 4 NAEP question.

Nevertheless, of the students responding to the question above, 51% of them answered incorrectly. Figure 5 is another example of this type of question from the NAEP, in response to which 52% of students selected an incorrect answer.

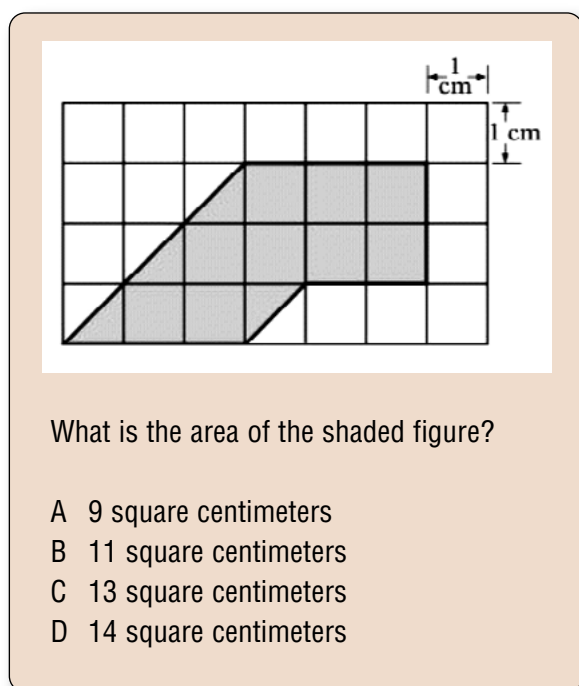


Figure 5. Grade 4 NAEP question.

Research focusing on the unit

From the literature and their performance on the NAEP, we know that students have difficulty developing an understanding of the unit concept in various dimensions. Work is needed to identify activities and/or materials that help students overcome those difficulties. The following activity is presented as an opportunity to give students an experience that will focus their attention on the unit rather than on the measurements they are attempting to calculate.

Suggested activity

The following activity has been extended and modified from the “Stuck on Stickers” activity that appears in NCTM’s *Navigating through Measurement in Grades 3–5* materials

(2005, pp. 58–61, 127–129). It can be used to investigate and build upon students’ understanding of units in one- and two-dimensions. This activity can be introduced as early as grades 3 or 4, however if students in upper grades have not had such an experience it is important they be exposed to similar opportunities.

PURPOSE: Students will develop understanding of the difference between linear and square units by considering the dimensions of various sticker sheets.

MATERIALS: 2 cm by 2 cm square stickers; string to go around sticker sheets; scissors.

PROCEDURE:

1. Tell students they will be forming sticker sheets of various sizes.
2. Inform them that they must create sticker sheets with the following dimensions: A: 6 cm \times 8 cm; B: 4 cm \times 14 cm; and C: 6 cm by 6 cm. Then have the students place a piece of string around the edge of the sticker sheet and measure the entire length of string needed to go around the sheet. This will help students focus on perimeter as a measurement that is linear or one-dimensional. This idea will be reinforced by having students extend the string in a straight line in order to measure its length. Recognising that area is two-dimensional will come from students realising they cannot simply relate the space covered by the string and the space covered by the stickers. This activity will reinforce that perimeter is the measurement around a figure whereas area is the measurement of the amount of space inside the figure.
3. Have them complete the table shown in Figure 6. The purpose of this table is for the students to organise their information. It will also allow them to make a direct connection between the length of string and the perimeter of the sticker sheet, as well as between the number of stickers and the area of the sheet.

Sticker Sheet	Length	Width	Length of String Around Perimeter	Number of Stickers	Area
A	6 cm	8 cm			
B	4 cm	14 cm			
C	6 cm	6 cm			

Figure 6. Recording table.

4. Ask the students the following questions in addition to the ones that appear in the *Navigating through Measurement in Grades 3–5* materials (2005, pp. 127–129):

- a. What is the difference between the units used for length, width, and perimeter versus those used for the area?

Possible answer: The units for the length and width consisted only of the edges of the stickers (two centimetres per sticker). The units for the area consisted of whole stickers (four square centimetres per sticker).

The purpose of this question is to help students realise that these measurements can be done along a straight line (linear or one-dimensional) whereas the area deals with the amount of space required to cover the figure. Eventually this can extend into how much material would be required to “fill-up” a three-dimensional figure.

- b. What is the difference between finding the length, width, and perimeter versus finding the area in relation to using the stickers?

Possible answer: The length, width and perimeter can be determined by seeing how much string is required to line up along the edge(s). The area can be determined by counting the number of stickers necessary to cover the entire section.

This gets the students to focus on the physical action they are doing to either determine the amount of string necessary to line up along the edge (one-dimensional) versus the number of stickers necessary to cover the figure. Students should begin to use words such as surround to discuss perimeter and cover to discuss area. These conversations will help student gain a deeper understanding of one- and two-dimensional units by considering perimeter and area.

Conclusions and recommendations

This activity is very brief and may seem straightforward to those who already have an understanding of one- and two-dimensional units. However, as the van Hiele model suggests, students must progress through the levels of geometric development sequentially. If certain activities or concepts are passed over with the assumption of understanding, students may only memorise the rules or formulas they must apply in order to calculate an answer. The activity described above would represent a more advanced topic at van Hiele’s visualisation level. It could help students develop conceptual understanding of the connections between one- and two-dimensional units. Such a foundation is crucial for students to understand measurements in different dimensions.

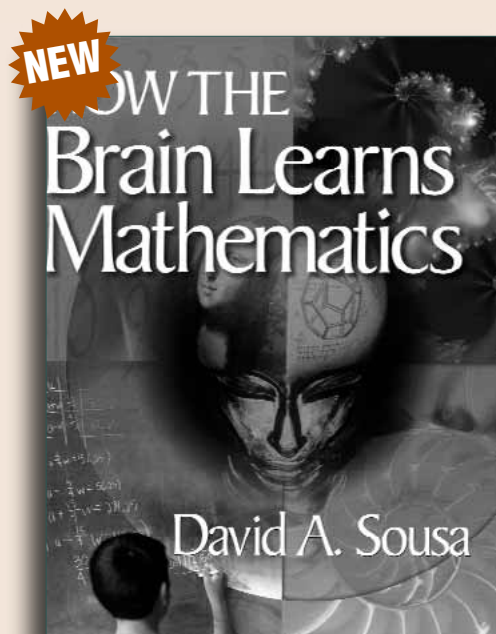
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